Calculus AB

5-4

The Exponential Function

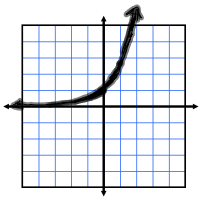
The Exponential Function - $e = 2.718281828\ldots$

$f(x) = e^x$

domain: $\mathbb{R}$

range: $(0, \infty)$

asymptotes: $y = 0$



The Derivative of the Exponential Function -

If $f(x) = e^x$, $f'(x) = e^x$

The Integral of the Exponential Function -

$\int e^x dx = e^x + C$

Solve for x to three decimal places. (pg 358)

2) $e^{\ln 2x} = 12$

$2x = 12$

$x = 6$ $\{6\}$

4) $4e^x = 83$

$e^x = \frac{83}{4}$

$\ln e^x = \ln \frac{83}{4}$

$x = \{3.033\}$

12) $\ln^2 x = 10$

$2 \ln x = 10$

$\ln x = 5$

$e^{\ln x} = e^5$

$x = \{148.413\}$

Find the derivative of each function.

40) $y = e^{-5x}$

$\frac{dy}{dx} = -5e^{-5x}$

54) $y = \frac{e^x - e^{-x}}{2}$

$\frac{dy}{dx} = \frac{e^x + e^{-x}}{2}$

Find the extrema and the points of inflection of the function.

84) $f(x) = x \cdot e^{-x}$

$F'(x) = e^{-x} - xe^{-x}$

$0 = e^{-x}(1 - x)$

$(1, e^{-1})$ $x = 1$ max

$F''(1) = e^{-1}(-2 + 1)$

$= -e^{-1} < 0$

$F'(x) = -e^{-x} - [e^{-x} - xe^{-x}]$

$F''(x) = -2e^{-x} + xe^{-x}$

$F''(x) = e^{-x}(-2 + x)$

$0 = e^{-x}(-2 + x)$

p.o.i @ 2

Day 1 - Assignment:
Pg. 358
1-15 odd,
39-75 odd,
79-85 odd.

Find or evaluate the integral. (pg 360)

old book 91) $-\frac{1}{2}\int -2xe^{-x^2}dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int e^u\,du$

$\boxed{-\frac{1}{2}e^{-x^2} + C}$

110) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

$u = e^x + e^{-x}$

$du = (e^x - e^{-x})\,dx$

$\int \frac{1}{u}\,du = \ln|e^x + e^{-x}| + C$

$\ln(e^x + e^{-x}) + C$

Solve the differential equation.

128) $\frac{dy}{dx} = \int (e^x - e^{-x})^2\,dx$

$y = \int (e^{2x} - 2e^x e^{-x} + e^{-2x})\,dx$

$= \int (e^{2x} + e^{-2x} - 2)\,dx$

$y = 2x$, $du = 2dx$: $\frac{1}{2}\int 2e^{2x}\,dx$ + $u = -2x$, $du = -2dx$: $-\frac{1}{2}\int -2e^{-2x}\,dx - \int 2\,dx$

$\frac{1}{2}\int e^u\,du - \frac{1}{2}\int e^u\,du - \int 2\,dx$

$\boxed{y = \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} - 2x + C}$

Day 2 - Assignment
Pg. 360
99 - 135 odd